

Power Analysis

PermutationTests.jl

Univariate Tests

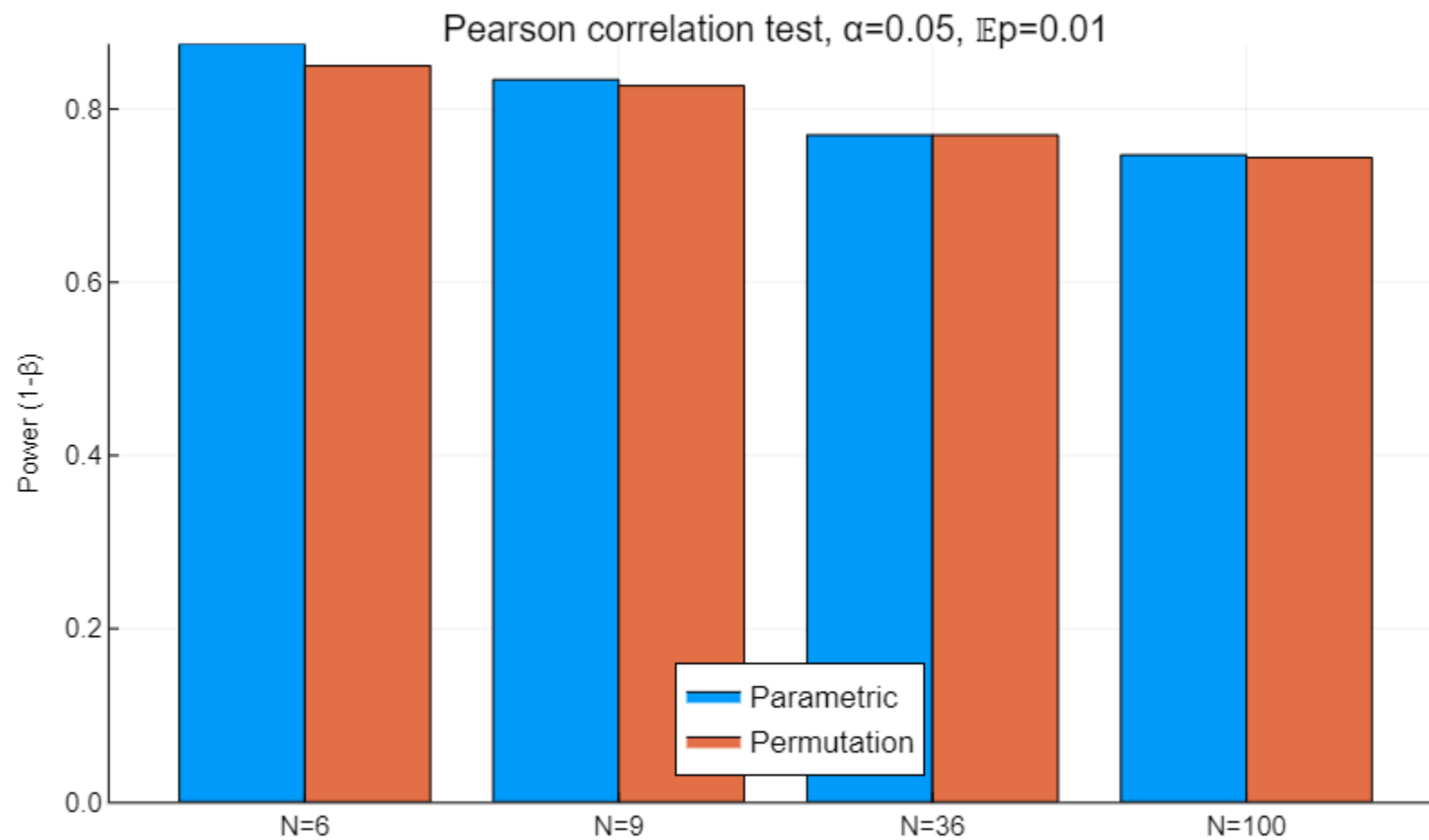
Power analysis with several sample sizes (group numerosities) N .

All power estimations are obtained with 1000 simulations.

Permutation tests are exact if the number of possible permutations is $< 1e5$, approximate otherwise.

Permutation tests are compared to the parametric counterpart.

$\mathbb{E}(p)$ is the expected p-value of the tests (effect size) using parametric tests

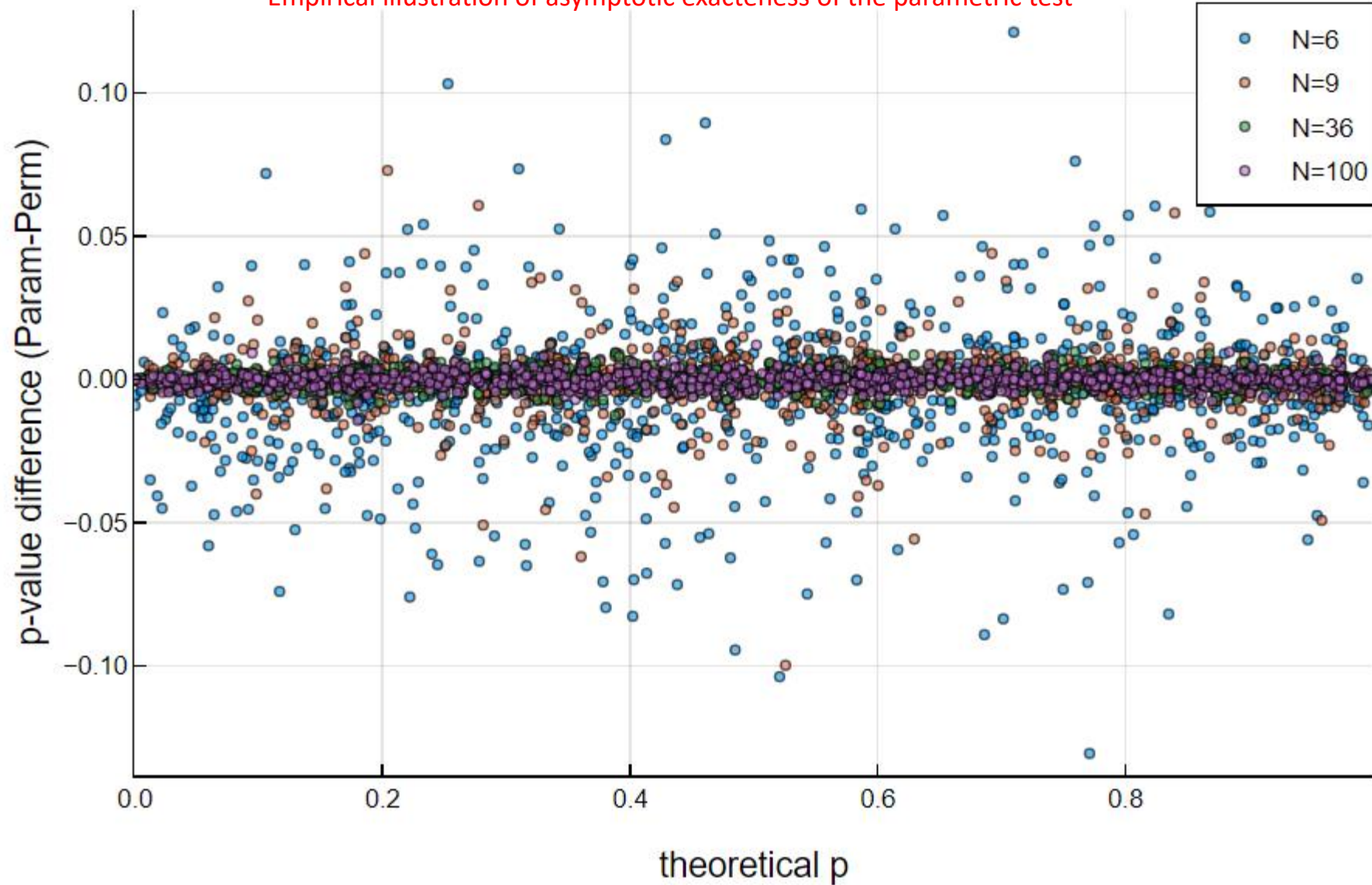


Similar power for permutation and parametric tests

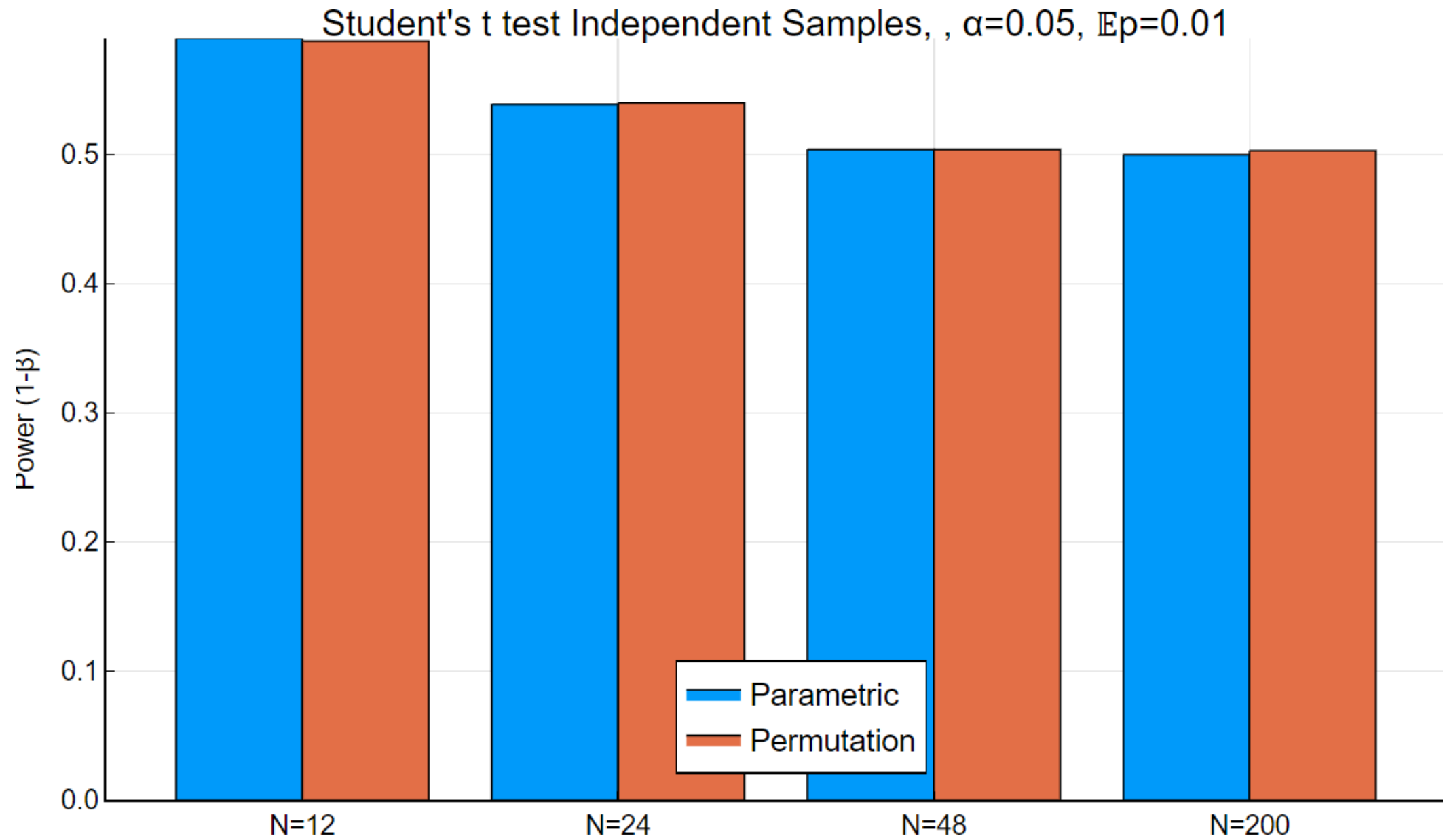
Difference of the p-values obtained by the parametric and permutation test at each simulation

Empirical illustration of asymptotic exactness of the parametric test

Correlation test



The difference is uniform across theoretical p-values and decreases as N (sample size) increases

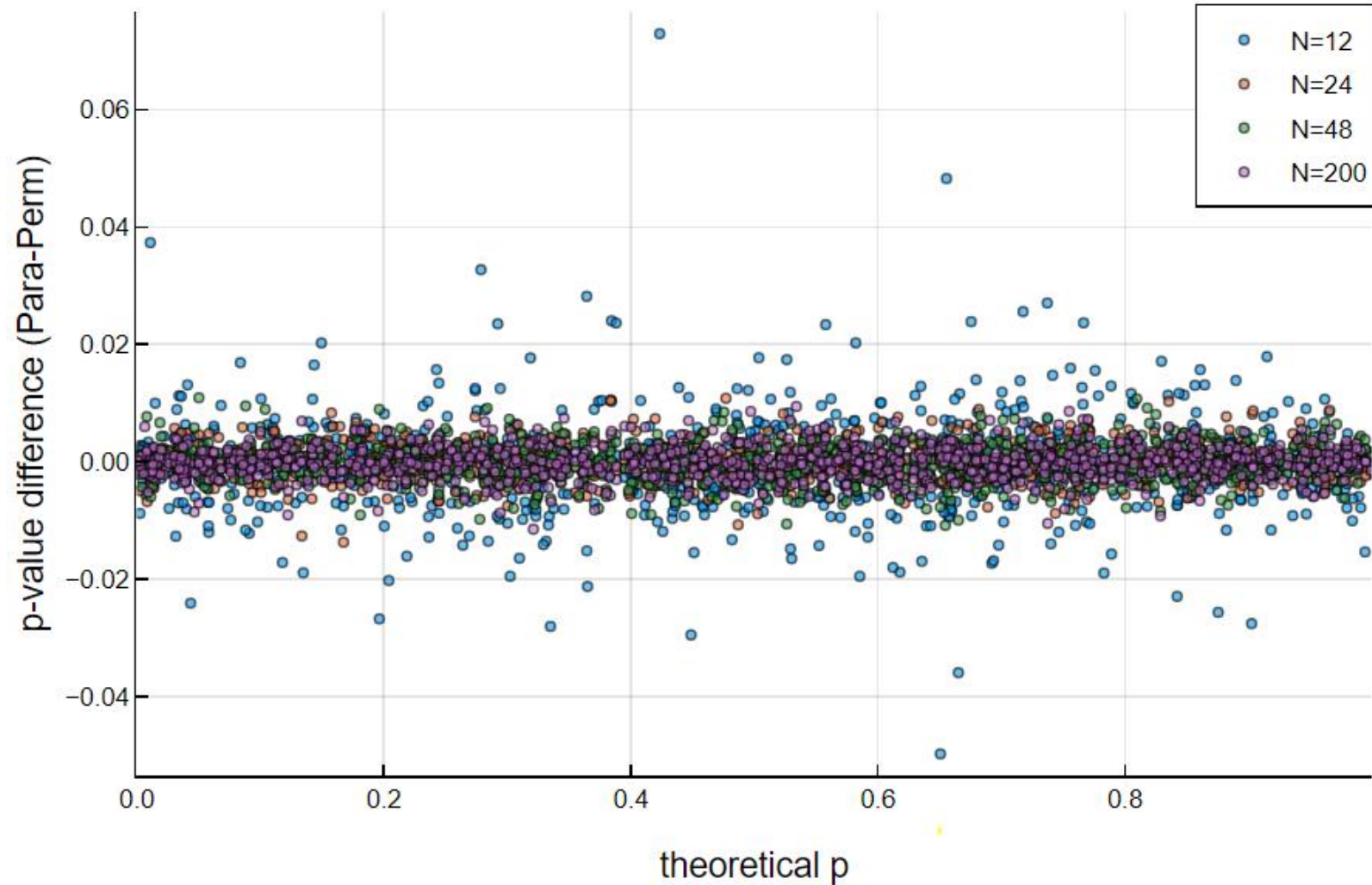


Similar power for permutation and parametric tests. Lower power as compared to the correlation test

Difference of the p-values obtained by the parametric and permutation test at each simulation

Empirical illustration of asymptotic exactness of the parametric test

Student's t test for Ind. Samples



The difference is uniform across theoretical p-values and decreases as N (sample size) increases

Multiple Comparisons Tests

100 simulations with various number of hypotheses (M) for a given number of observations (N)

Correlated hypothesis are generated under H_0 and H_1 .

The FWE is fixed at 0.05

Permutation tests are exact if the number of possible permutations is $< 1e5$,
approximate otherwise.

Correlation test:

We test the correlation between a variable x and M variables Y .

Random $(M+1)$ -multivariate Gaussian data is generated with expected correlation matrix such that:

- the expected correlation among the Y variables = the value yielding a p-value of 0.1 by a parametric test
- the expected correlation between x and all Y variables = the value yielding a p-value of 0.01 by a parametric test with the chosen sample size, i.e., all hypotheses are generated under H_1 .

Correlation tests, 100 simulations

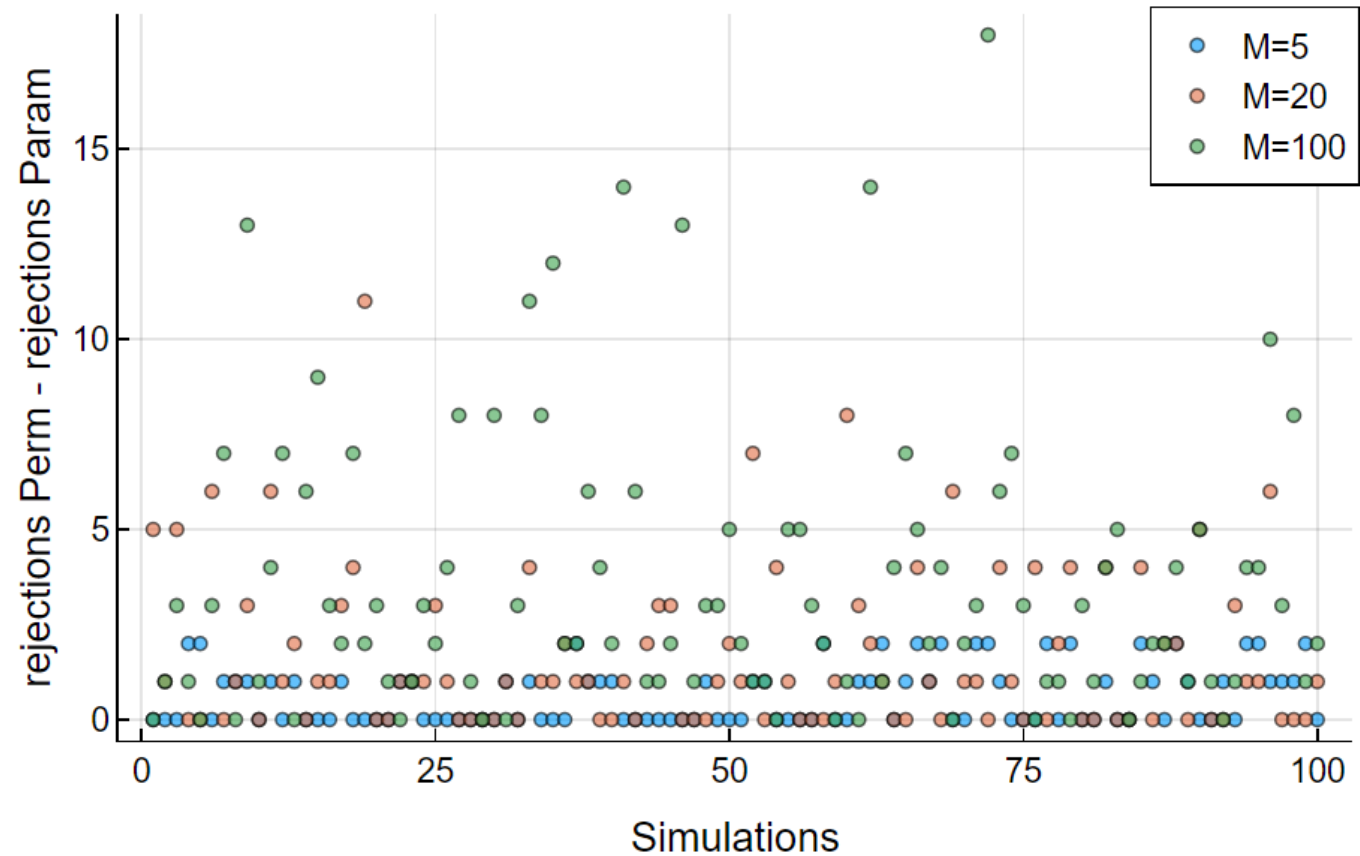
Average rejections / hypotheses under H1 (Power)

correlation test, 24 observations

M	Permutation	parametric
5	0.59	0.47
20	0.37	0.28
100	0.18	0.15

On the average of all simulations the permutation max-r test is always more powerful than the Bonferroni correction procedure. The difference tends to decrease as the number of hypotheses (M) increases

Correlation tests, 100 simulations



The permutation max-r test is always more powerful or as powerful as the Bonferroni correction procedure (for each simulation the difference of the rejection is always positive)

Student's t test for independent samples:

We test the difference of means of M pairs of variable variables (x, y) .

Let X and Y be the collection of the M x and y variables, respectively.

Random (M) -multivariate Gaussian data is generated independently for both X and Y with expected correlation matrix such that the expected correlation ρ among all M variables = the value yielding a p-value of 1, 0.1 and 0.01 (three sets of simulations) by a parametric test

50% of the hypothesis are generated under H_0 and 50% under H_1 .

For the hypotheses generated under H_1 , the shift given to mean of the y variable is such that the p-value of a parametric t-test is expected to be 0.01 given the chosen sample size.

Student's t tests for independent samples, 100 simulations

Average rejections / hypotheses under H1 (Power)

Student's t test for Ind. Samples, [8, 8] subjects per group

M	Permutation	Parametric
5	0.27	0.26
20	0.13	0.12
100	0.04	0.04

5	0.34	0.32
20	0.12	0.10
100	0.07	0.05

5	0.39	0.31
20	0.19	0.11
100	0.05	0.02

$$\mathbb{E}p(\rho) = 1$$

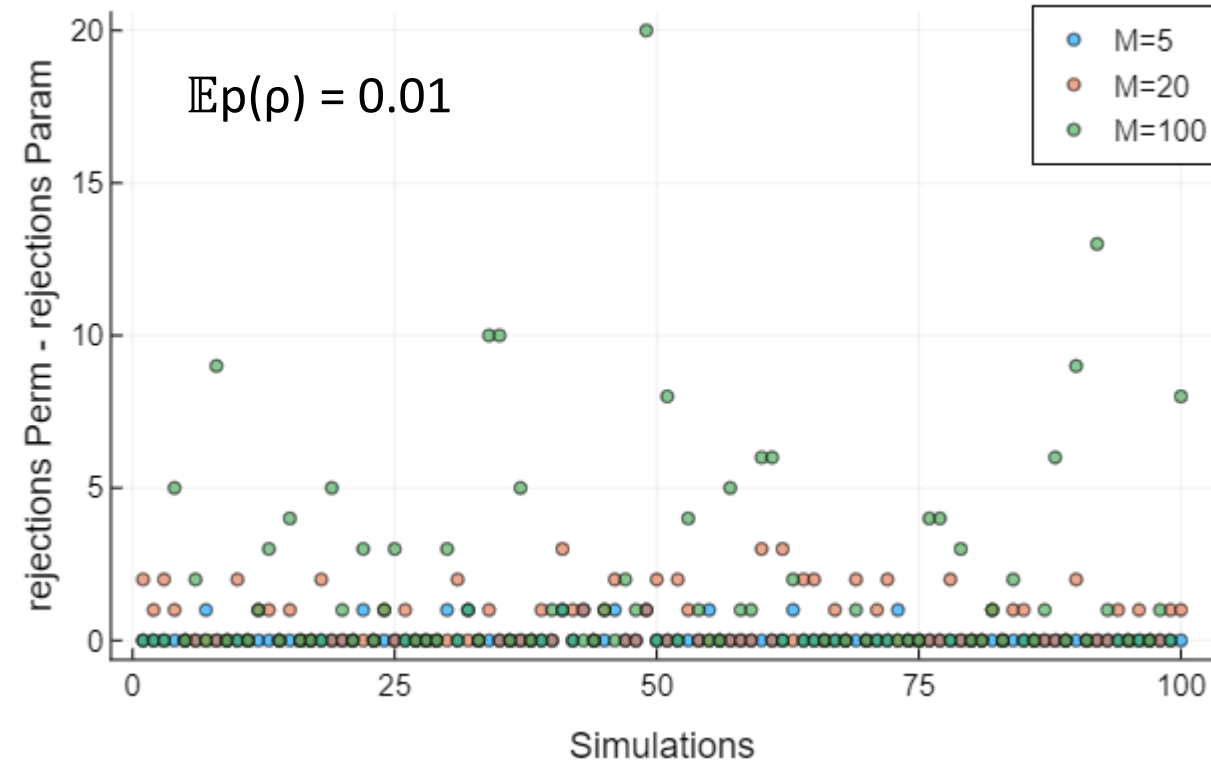
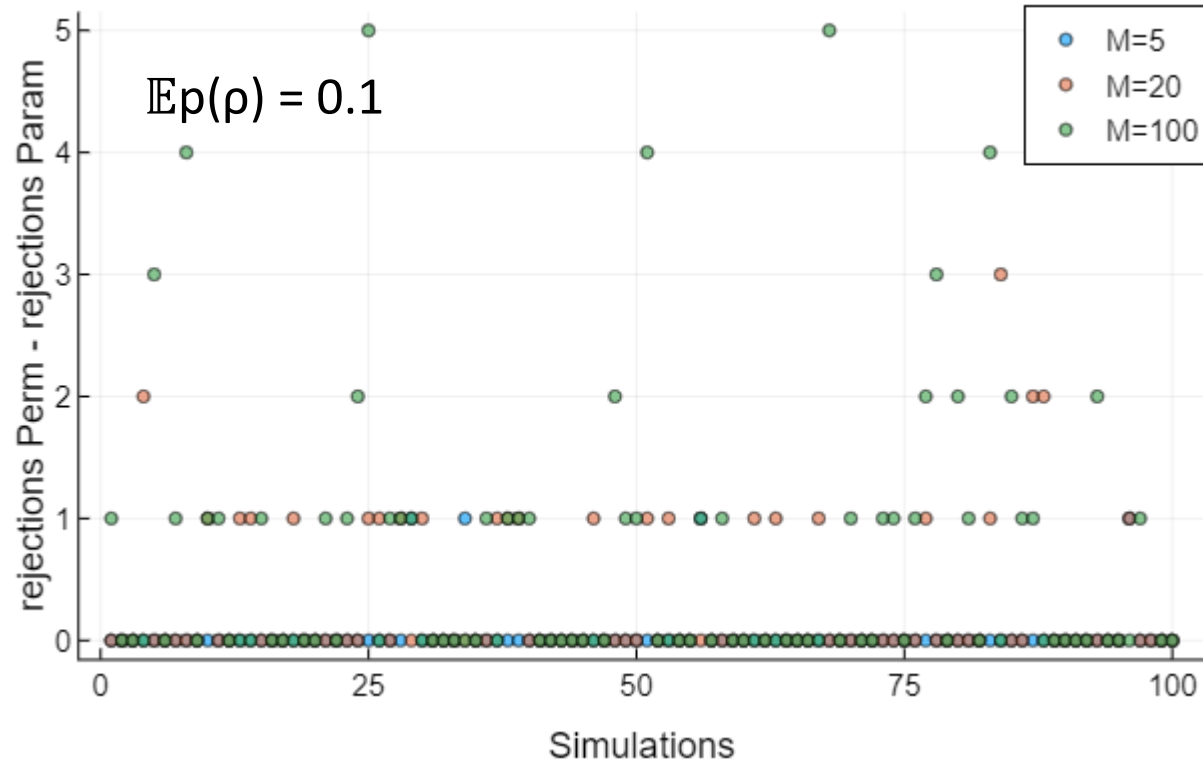
$$\mathbb{E}p(\rho) = 0.1$$

$$\mathbb{E}p(\rho) = 0.01$$

On the average of all simulations the permutation max-t test is always more powerful than the Bonferroni correction procedure.

The advantage of the permutation approach increases as the correlation among variables (ρ) increases.

Student's t tests for independent samples, 100 simulations



The permutation max-t test is always more powerful or as powerful as the Bonferroni correction procedure (for each simulation the difference of the rejection is always positive).

The advantage of the permutation approach increases as the correlation among variables (ρ) increases.